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Instabilities and particle production in S-brane geometries

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Abstract: We study the classical stability of a class of S-brane geometries having cosmological horizons. By considering the perturbations of the metric in these geometries we establish that their horizons are unstable in the sense that an observer trying to cross the horizon experiences an infinite flux of radiation at the instant of crossing. The backreaction of this radiation is likely to convert the horizons into curvature singularities, similar to the instability of the internal Cauchy horizon of the Reissner-Nordström black hole. We also compute the particle production by the time-dependent fields in the future regions of these geometries, and find that the spectrum of produced particles is thermal, with temperature coinciding with the Hawking temperature computed by euclideanizing the metric in the static region. Possible implications of these results are discussed.

Keywords: p-branes, Black Holes, Cosmology of Theories beyond the SM.

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Contents

1. Introduction

S-branes are spacelike surfaces in spacetime along which transitions between different vacua are speculated to take place in string theory [1]. Their study is motivated by the desire to be able to extend the present theoretical toolbag to include techniques for understanding time-dependent problems in string theory. If the transitions described by S-branes really do arise, parts of their behavior should be describable in terms of the evolution of the low-energy fields of supergravity, perhaps also with a rolling tachyon or tachyons which describe the field-theory version of the transition between vacua.

Several time-dependent supergravity solutions have been proposed as corresponding to S-branes [2]–[6]. Although many have an FRW-like singularity at early times, some do not and instead have an interesting global structure consisting of non-singular and asymptotically-flat past and future regions separated by static regions having time-like singularities. Some of these geometries have been studied in fair detail. In particular, ref. [6] performs a general analysis of the charge, tension, entropy and Hawking temperature of many of these spaces. It was concluded there that the time-dependent regions provide an interesting interpretation that fits very well with the original S-brane proposal [1]. The static regions could be interpreted as the fields external to a pair of oppositely-charged, negative-tension branes. A similar interpretation was proposed earlier in terms of orientifold planes [7].

Our purpose here is to explore two important issues of stability for these geometries, which were not completely addressed in ref. [6]. The first of these is the classical stability of the solutions. Our preliminary study showed that small perturbations in Klein-Gordon scalar fields in these geometries do not grow with time, but *do* infinitely blue-shift for inertial observers passing through the horizons. This signals a potential instability, because the backreaction of this energy density on the metric is ultimately likely to convert the horizons into curvature singularities [10]. Here we extend this observation somewhat by performing a similar analysis for the modes of the metric itself, adapting for this purpose the methods applied by Chandrasekhar and Hartle to the Reissner-Nordström black hole.

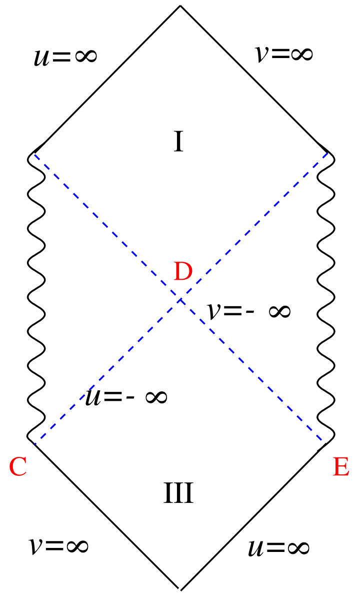

Figure 1: Penrose diagram for the study of stability.

We arrive at the same conclusion as before, thereby strengthening the arguments that the horizons of this geometry are *unstable*. This fact indicates that gravitational perturbation will generally introduce *null-like singularities* in the geometry.

The presence of these singularities makes these solutions more similar in character to the general S-brane solutions discussed in [5], for which similar curvature singularities are already present on null-like surfaces at the classical level. Only for special values of some parameters do the singular surfaces of [5] become horizons, and based on our calculation one might wonder if these horizons are stable to the formation of singularities once metric fluctuations are considered.

The second question we address concerns the particle production which is produced by the non-static metric at late times. In ref. [6] a Hawking temperature was associated with the static regions of the metric, by requiring in the usual way that there be no conical singularities in their euclidean sections. This temperature was argued to be related to the particle spectrum which is seen by the (accelerating) static observers. Here we directly compute the particle production in the non-static region and show that it is also thermal in character, with the same temperature that was found previously for the static regions.

2. Classical instability

In this section we present in detail the stability analysis for gravitational metric perturbations of the simplest S0-brane geometry of ref. [6]. We will closely follow a procedure used by Chandrasekhar and Hartle [8, 9], to show the instability of the Reissner-Nordström (RN) metric. We show that the same conclusion also applies to the S0-brane geometries of interest here.

We start by reviewing the preliminary analysis performed in ref. [6], where the Klein-Gordon modes were analysed arriving to the preliminary conclusion of instability of the Cauchy horizon. We then perform a more rigorous analysis by considering in detail the metric perturbation modes, arriving to the same conclusion of instability.

Consider then the simplest geometry that corresponds to an uncharged S0-brane in 4 dimensions, whose Penrose diagram is given in figure 1, and is simply a $\pi/2$-rotation of the Penrose diagram for the Schwarzschild black hole. (The analysis for a charged brane can be done in analogy with the present case, with virtually no modification to the equations we present here, and with the same results.) The metric for this spacetime is:

$$ds^2 = -\left(1 - \frac{2p}{t}\right)^{-1} dt^2 + \left(1 - \frac{2p}{t}\right) dr^2 + t^2 d\theta^2 + t^2 \sinh^2\theta \; d\phi^2 \,, \tag{2.1}$$

where

$$h(t) = 1 - \frac{2p}{t} \,. \tag{2.2}$$

The coordinate t is a time coordinate in regions I and III of figure 1, but is a spatial coordinate in the other two regions.

The coordinates of eqs. (2.1) and (2.2) break down on the surfaces $t = 2p$, which correspond to the diagonal lines which form the boundaries of regions I and III. These are the horizons for this geometry. From the Penrose diagram it is easy to see that the line CDE is a Cauchy horizon, since the initial-value problem in region III does not uniquely determine the field evolution at points to the future of this line. Field evolution past the line CDE is not unique because it can also be influenced by signals from the time-like singularities.

The Klein-Gordon equation for a massive scalar field propagating in the background eq. (2.1), is given by

$$-\frac{1}{\sqrt{g}} \partial_M \left[\sqrt{g} g^{MN} \partial_N\right] \psi + M^2 \psi = 0$$

in the time-dependent regions I and III. Now, since we are interested in the near horizon limit of the modes, it is convenient to write the KG equation in terms of isotropic coordinates defined as $\tau = t - 2p$; the equation is then given by

$$-\frac{1}{\sqrt{g}} \partial_\tau \left[\sqrt{g} g^{\tau\tau} \partial_\tau\right] \psi - g^{rr} \partial_r^2 \psi - \frac{1}{\tau^2 H_+ \sqrt{h}} \partial_i \left[\sqrt{h} h^{ij} \partial_j\right] \psi + M^2 \psi = 0 \,. \tag{2.3}$$

Here, for clarity, we denote $h_{ij}(\theta, \phi)$ for the metric on the 2-dimensional maximally-symmetric hyperbolic space, and write $g_{ij}(\tau, \theta, \phi) = \tau^2 H_+ h_{ij}(\theta, \phi)$, where $H_+ = 1 + 2p/\tau$. The relevant metric components are: $g_{\tau\tau} = -H_+$ and $g_{rr} = H_+^{-1}$. The functional form of the metric involved permits separation of variables, so we take $\psi(r, \tau, \theta, \phi) = e^{i\sigma r} f(\tau) \, L_k(\theta, \phi)$, where σ and k are separation constants determined by the eigenvalue equations:

$$-\partial_r^2 e^{i\sigma r} = \sigma^2 e^{i\sigma r} \qquad \text{and} \qquad -\frac{1}{\sqrt{h}} \partial_i \left[\sqrt{h} h^{ij} \partial_j\right] L_k = k^2 L_k \,.$$

Both eigenvalue equations can be solved explicitly, and delta-function normalizability of the solutions require both $\sigma^2 \geq 0$ and $k^2 \geq 0$. The temporal eigenvalue equation then becomes:

$$-\frac{1}{\sqrt{g}}\frac{d}{d\tau}\left[\sqrt{g}g^{\tau\tau}\frac{df}{d\tau}\right] + \left[g^{rr}\sigma^2 + \frac{k^2}{\tau^2 H_+} + M^2\right] f = 0\,. \tag{2.4}$$

Near the horizon, $\tau \to 0$ and the asymptotic form is governed by the limits $H_+ \to 2p/\tau$. The metric functions therefore reduce to $g_{\tau\tau} \to \alpha_\tau \tau^{-1}$, $g_{rr} \to \alpha_r \tau$ and $\omega \to \alpha_\omega$. The precise values of the constants α_τ, α_r and α_ω are not required, apart from the following ratio: $\frac{\alpha_\tau}{\alpha_r} = (2p)^2$. With these limits, the Klein-Gordon equation becomes, in the near-horizon limit:

$$\ddot{f} + \frac{1}{\tau}\dot{f} + \left[\frac{\alpha_\tau \sigma^2}{\alpha_r}\frac{1}{\tau^2} + \alpha_\tau \tau^{-1}\left(M^2 + \frac{k^2}{\alpha_\omega^2}\right)\right] f = 0\,. \tag{2.5}$$

If $\sigma \neq 0$, then the solutions are oscillatory, having the form $f \sim \tau^{a_0}$, with $a_0 = \pm i\sigma\sqrt{\alpha_\tau/\alpha_r}$. If $\sigma = 0$, then a similar argument shows that the solutions are nonsingular as $\tau \to 0$.

One can now estimate whether an instability does exist by computing the energy, $E = -u^m \partial_m \psi$ of the Klein-Gordon modes considered above, as seen by an observer whose velocity, $u = M\partial_t + N\partial_r$, is well-behaved as it crosses the horizon. The normalization condition $u^2 = -1$ in the vicinity of the horizon allows a determination of how M and N must behave as $\tau \to 0$ (in isotropic coordinates) in order to remain non-singular. We find in this way $u^2 \sim -\alpha_\tau M^2 \tau^{-1} + \alpha_r N^2 \tau$, which is regular near $\tau \to 0$ provided $M \sim \tau^{1/2}$ and $N \sim \tau^{-1/2}$ near the horizon. With this choice, one then finds

$$-E = M\partial_\tau \psi + N\partial_r \psi \sim \psi \tau^{-1/2}\,. \tag{2.6}$$

Using the asymptotic solution found below eq. (2.5): $\psi \sim \tau^{a_0}$ with $a_0 = \pm i\sigma\sqrt{\alpha_\tau/\alpha_r}$, we see that $E \to \infty$ as the horizon is approached. This suggests that the stress-energy density of the mode under consideration diverges as well in this limit. As such, this mode is likely to destabilize the metric modes near the past horizon.

We will now confirm this result by a full analysis of the metric perturbations. For later purposes it is convenient to define several quantities. We define the surface gravity of the solution to be

$$\kappa_0 = \frac{1}{2}\left|\frac{dh(t)}{dt}\right|_{t=2p}, \tag{2.7}$$

and the 'tortoise' coordinate as

$$t_* \equiv \int \frac{dt}{h(t)} = t + \frac{1}{2\kappa_0}\ln|t - 2p|\,, \tag{2.8}$$

where $-\infty < t_* < +\infty$ corresponds to the range $2p < t < \infty$. Notice that t_* increases from past to future in region I, but decreases from past to future for region III.

Our focus is on metric perturbations in region III of the Penrose diagram, which is to the past of the Cauchy horizon. We also focus on the 'axial' perturbations of the metric, which are defined as follows [9]: Take one of the angular coordinates ϕ and change its line element by

$$d\phi^2 \to (d\phi - q_1 dr - q_2 dt - q_3 d\theta)^2\,, \tag{2.9}$$

where the q_i's are arbitrary functions of r, t and θ. The metric perturbations can be written, when specialized to the 'axial' modes, as a scalar equation for the field, $\Phi(t,r,\theta) = t^2 h(t)\,(q_{2,\theta} - q_{3,t})\sinh^3\theta$ (see [9] for details). Given the symmetries of the problem, the field equation for Φ can be solved by separation of variables, with the field $\Phi(t,r,\theta)$ decomposed as

$$\Phi(t,r,\theta) = t^{-1} Z(t)\,\Theta_k(\theta)\,e^{i\sigma r}\,, \tag{2.10}$$

with the functions $\Theta_k(\theta)$ and $e^{i\sigma r}$, defined as solutions to the following eigenvalue equations [9]:

$$-\frac{\partial^2}{\partial r^2}\,e^{i\sigma\,r} = \sigma^2 e^{i\sigma\,r}\,, \qquad \text{and} \qquad \sinh^3\theta\frac{d}{d\theta}\left(\frac{1}{\sinh^3\theta}\frac{d}{d\theta}\Theta_k\right) = k^2\Theta_k\,.$$

Writing the relevant equations in terms of the tortoise coordinate defined above, the temporal eigenvalue equation then becomes:

$$\frac{d^2 Z(t_*)}{dt_*^2} + \sigma^2 Z(t_*) = V(t(t_*))Z(t_*)\,, \tag{2.11}$$

where the potential in the previous equation is given by

$$V(t) = -\frac{(t-2p)}{t^3}\left[\frac{2p}{t} - 2\,h(t) + k^2\right]. \tag{2.12}$$

Our interest is in the behavior of the mode Φ near the horizon in region III ($t_* \to -\infty$), given its form in the asymptotic past ($t_* \to \infty$). We therefore need the asymptotic behavior of the potential in these limits, which is

$$V(t_*) \propto \begin{cases} 1/t^2 & \text{for } t_* \to t \to \infty \\ e^{2\kappa_0 t_*} & \text{for } t_* \to -\infty. \end{cases} \tag{2.13}$$

Since the potential falls off faster than $1/t_*$, the asymptotic behavior of the solutions of (2.11) for $t_* \to \pm\infty$ is given by $e^{\pm i\sigma t_*}$.

Our initial condition must have only incoming waves in the asymptotic past, which for region III means for $t \to +\infty$. Given this we wish to compute the coefficients $A(\sigma)$ and $B(\sigma)$ which control the behavior of the solutions near the horizon, according to

$$Z(t_*) \to \qquad e^{i\sigma t_*} \qquad t_* \to t \to \infty \tag{2.14}$$

$$\to A(\sigma)\,e^{-i\sigma t_*} + B(\sigma)\,e^{i\sigma t_*} \qquad t_* \to -\infty\,. \tag{2.15}$$

(These boundary conditions look slightly odd compared to the Black Hole case, due to the reversal of roles between $+\infty$ and $-\infty$ which may be traced to the unconventional property that t_* becomes more negative into the future for region III (see figure 1). This difference of boundary conditions represents a key difference between the S0-brane geometry of interest here and other similar geometries, like the Reissner-Nordström metric.)

In this way, the problem is reduced to that of scattering of waves on the potential of eq. (2.12). In order to distinguish transmission from reflection for a given initial wave we must distinguish the edges CD and DE of the Penrose diagram. In order to do so, we restore the r-dependence, $e^{i\sigma r}$, of the initial wave, corresponding to a wave whose wavefronts initially move towards increasing r. It is convenient to express the solutions in terms of the null-like coordinates $u = t_* + r$, $v = t_* - r$, in which case the initial configuration is $Z(t_*, r) \to e^{i\sigma u}$ for $t \to \infty$, while the near-horizon limit of eq. (2.15) becomes

$$Z(t_*, r) \to e^{i\sigma u} + [B(\sigma) - 1]\, e^{i\sigma u} + A(\sigma)\, e^{-i\sigma v} \,. \tag{2.16}$$

From the previous expression (or from looking at the figure) it is clear that the transmitted part of the wave will cross the edge CD, while the reflected part crosses edge DE.

If we take a general, properly weighted, initial amplitude $W(\sigma)$ then the above formulae can be rewritten as

$$Z(t_*, r) \to X(v) + Y(u) \qquad (u,\, v \to -\infty)\,, \tag{2.17}$$

where

$$X(v) = \int_{-\infty}^{\infty} W(\sigma)\, A(\sigma)\, e^{-i\sigma v} d\sigma\,, \tag{2.18}$$

$$Y(u) = \int_{-\infty}^{\infty} W(\sigma)\, [B(\sigma) - 1]\, e^{i\sigma u} d\sigma\,. \tag{2.19}$$

To argue for instability we now compute the energy contained in the radiation as seen by a radially-moving inertial observer crossing the Cauchy horizon. The four velocity, $\mathbf{U}$, of such an observer is given by [6]:

$$U^r = \frac{dr}{d\tau} = \frac{E}{h(t)}\,; \qquad U^{t_*} = \frac{dt_*}{d\tau} = -\frac{1}{h(t)}[E^2 - h(t)]^{1/2}\,, \qquad U^i = 0\,, \tag{2.20}$$

where τ is the proper time and we choose the negative sign for U^{t_*} in region (III) so that $\mathbf{U}$ is future-directed. Notice that the integration constant, E, which labels the observer's geodesic can be negative.

A measure of the energy of the fluctuation Φ as seen by this observer is given by $\mathcal{F}$, defined by

$$\mathcal{F} = U^\mu Z_{,\mu} = U^r Z_{,r} + U^{t_*} Z_{,t_*}\,, \tag{2.21}$$

or:

$$\mathcal{F} = \frac{1}{h(t)}\left[E Z_{,r} - (E^2 - h(t))^{1/2} Z_{,t_*}\right]. \tag{2.22}$$

In terms of X and Y, in the near-horizon limit we have

$$\begin{aligned} Z_{,r} &\to X_{,-v} + Y_{,u}\,, \\ Z_{,t_*} &\to -X_{,-v} + Y_{,u}\,, \end{aligned} \tag{2.23}$$

and so eq. (2.22) becomes

$$\mathcal{F} \to \frac{1}{h}\left[X_{,-v}\left(E + [E^2 - h]^{1/2}\right) + Y_{,u}\left(E - [E^2 - h]^{1/2}\right)\right]. \tag{2.24}$$

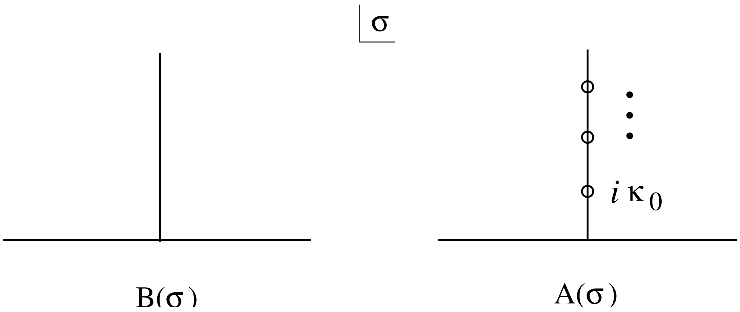

Figure 2: Domains of analyticity of $A(\sigma)$ and $B(\sigma)$.

We now ask whether $\mathcal{F}$ diverges on one of the horizons, CD or DE. On CD, v remains finite, while $u \to -\infty$. This means that, for $E < 0$, the term involving $X_{,v}$ of (2.24) remains finite while the term with $Y_{,u}$ could diverge. Hence:

$$\mathcal{F}_{CD} \to -4p\,|E|Y_{,u}e^{-\kappa_o u} \qquad (u \to -\infty \quad \text{on} \quad CD)\,. \tag{2.25}$$

On DE, by contrast, it is u which remains finite, while $v \to -\infty$. In this case, when $E > 0$, the term involving $Y_{,u}$ remains finite but the term with $X_{,-v}$ diverges. Hence

$$\mathcal{F}_{DE} \to 4p\,EX_{,-v}e^{-\kappa_o v} \qquad (v \to -\infty \quad \text{on} \quad DE)\,. \tag{2.26}$$

From these expressions we see that the existence of a divergence in $\mathcal{F}$ depends on the behavior of

$$X_{,-v} = \int_{-\infty}^{\infty} W(\sigma)\, i\sigma\, A(\sigma)\, e^{-i\sigma v}\, d\sigma\,, \tag{2.27}$$

$$Y_{,u} = \int_{-\infty}^{\infty} W(\sigma)\, i\sigma\, [B(\sigma) - 1]\, e^{i\sigma u}\, d\sigma\,, \tag{2.28}$$

near the horizons. The integrals may be evaluated by contour integration, with the contour closed in the *lower* half-plane for $Y_{,u}$ (where our interest is in $u \to -\infty$) and in the *upper* half plane for X_{-v} (where we care about $v \to -\infty$). The result depends on the singularities of $A(\sigma)$ and $B(\sigma)$, which have analytic properties which can be quite generally determined using the arguments of Chandrasekhar and Hartle [8, 9] with virtually no modification. One finds in this way the function $A(\sigma)$ is analytic on the upper half plane except for the poles located at $in\kappa_0$, where n is a natural number, while the function $B(\sigma)$ is analytic on the entire upper half plane. This singularity structure is illustrated in figure 2.

Applying these results to the flux on surface DE, we evaluate the integral by deforming the contour into the upper half plane. The dominant contribution is then given by the closest pole to the real axis, which occurs at $\sigma = i\kappa_0$. Then the integral giving $X_{,-v}$ becomes

$$X_{,-v} \propto e^{\kappa_0 v}\,. \tag{2.29}$$

We see, from (2.26), that $\mathcal{F}_{DE}$, is therefore bounded as the observer crosses the horizon.

Similarly evaluating the integral for $Y_{,u}$ by closing the contour in the lower half-plane, we find contributions only from the real axis: $i\sigma = 0$. Consequently, for $W(\sigma)$ analytic on the real axis, $Y_{,u}$ is $O(1)$ near the horizon CD. It then follows from (2.25) that the flux $\mathcal{F}_{CD}$ at CD necessarily *diverges*.

Although we have derived the instability for the lower Cauchy horizons, we also expect the same conclusion to hold for the upper horizons which bound region I of the Penrose diagram. We can argue this on grounds of continuity, as we follow the energy seen by a family of observers who cross the horizons in the vicinity of point D of the Penrose diagram. We expect from this that the same infinite blue-shift seen by inertial observers for horizons CD and DE also extends to the other two horizons.

This calculation confirms the preliminary work of ref. [6], which argued for infinite energy for Klein-Gordon modes on this metric. The present calculation shows that the same conclusion also holds for *bona fide* metric modes. The existence of this divergent energy strongly suggests that the horizon is unstable towards becoming a curvature singularity, due to the metric's back-reaction of these large energy densities: gravitational perturbations will introduce null-like singularities in the geometry. This conclusion strongly resembles the same results for the instability of the RN black hole.

The previous calculation can be extended to more general asymptotically-flat time-dependent backgrounds representing cosmological horizons. In general, the temporal eigenvalue equation that rules the perturbations will take the form of a Schrödinger equation like eq. (2.11). When the resulting potential is characterized by an asymptotic behavior given by (2.13), the resulting cosmological horizon will be unstable under cosmological gravitational perturbations. Also, the analysis of 'polar' perturbations gives similar results as does the axial case presented above [9].

3. Particle production

We now turn our attention to the time-dependent region (region I) which is to the future of all of the horizons. Because the metric in this region is not static, it should cause particle production for any quantum fields which propagate within it. We compute this particle production here, and show that it has features which resemble a thermal distribution whose temperature is given by the Hawking temperature, as defined in ref. [6] for the *static* part of the metric. We find in this way a connection between the properties of the time-dependent region I and the static regions which are separated from it by the horizons. These did not *a priori* need to be related, and such a relation seems even more odd if the surface dividing these regions represents a curvature singularity rather than a horizon.

To this end we again consider the simplest non-trivial case, namely the S0-brane in four dimensions, with metric given by (2.1),

$$ds^2 = -\left(1 - \frac{2p}{t}\right)^{-1} dt^2 + \left(1 - \frac{2p}{t}\right) dr^2 + t^2 d\theta^2 + t^2 \sinh^2\theta \, d\phi^2 \,. \tag{3.1}$$

Let us first recall the formal calculation of the Hawking temperature performed in [6]. This requires the cancellation of a conical singularity, that can be obtained by requiring the proper periodicity for the euclidean time in the euclidean section of the metric's static regions. In the near-horizon limit the euclidean metric for the static regions takes the form:

$$ds_E^2 = dR^2 + \kappa_0^2 R^2 d\tau^2 + \cdots \,, \tag{3.2}$$

where $R^2 = (2p - r)/2p$ and $\kappa_0 = 1/(4p)$ denotes the surface gravity at the horizon, as in the previous section. Demanding no conical singularity at the horizon ($R = 0$) requires the euclidean time coordinate τ to be periodic $\tau \sim \tau + 2\pi/\kappa_0$, leading to the Hawking temperature:

$$T_H = \frac{\kappa_0}{2\pi} = \frac{1}{8\pi p}\,. \tag{3.3}$$

Ref. [6] argued this temperature to be interpretable as the temperature of the particle distribution seen by static particle detectors in the static regions.

We now compute a logically unrelated quantity: the particle production of a massless Klein-Gordon field caused by the time-dependent fields in region I. To this aim we consider the massless Klein-Gordon equation:[1]

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\right)\Phi = 0\,, \tag{3.4}$$

which we again solve by separating variables to obtain the following mode functions: $u_{m\sigma k}(t,r,\theta,\phi) = e^{i(m\phi+\sigma r)}\ \Theta_{km}(\theta)\ \Omega(t) + \text{c.c.}$ Here the integer m and real quantities σ and k are the quantum numbers associated with the ϕ, θ and r coordinates, respectively. We obtain the following θ dependence:

$$\Theta_{km}(\theta) = aQ_m^{\frac{1}{2}(\sqrt{1+4k^2}-1)}(\cosh\theta) + bP_m^{\frac{1}{2}(\sqrt{1+4k^2}-1)}\,(\cosh\theta)\,, \tag{3.5}$$

where a and b are integration constants. $P_m^r(x)$ and $Q_m^r(x)$ are the usual associated Legendre functions. In what follows we restrict our analysis to the simplest case, $m = k = 0$.

The physics of interest lies in the t-dependent part, which can be usefully rewritten as a Schrödinger-like equation by performing the substitution

$$\Omega(t) = F(t)\left[\frac{p}{\sqrt{t\,(t-2p)}}\right]. \tag{3.6}$$

It is also convenient to perform a change of independent variable $x = t - 2p$ in order to place the horizon at $x = 0$. With these choices the t-dependent equation takes the form:

$$\frac{d^2F(x)}{dx^2} - V(x)\,F(x) = 0 \tag{3.7}$$

with

$$-V(x) = \left(\frac{\sigma^2(x+2p)^2}{x^2} + \frac{2(x+p)}{x(x+2p)^2} + \frac{2(x+p)p}{x^2(x+2p)^2} - \frac{1}{x(x+2p)} - \frac{(x+p)^2}{x^2(x+2p)^2}\right). \tag{3.8}$$

In order to solve this equation we replace $V(x)$ with an approximate potential, $V_{\text{approx}}(x)$, which is chosen to properly reproduce the asymptotic form of $V(x)$ as $x \to \infty$. (This is similar in spirit to replacing $V(x)$ with one of its Padé approximants.) For these purposes we choose

$$-V_{\text{approx}}(x) = \left(1 + \frac{4p}{x} + \frac{4p^2}{x^2}\right)\sigma^2\,. \tag{3.9}$$

[1]See [11] for the analysis of the black hole case.

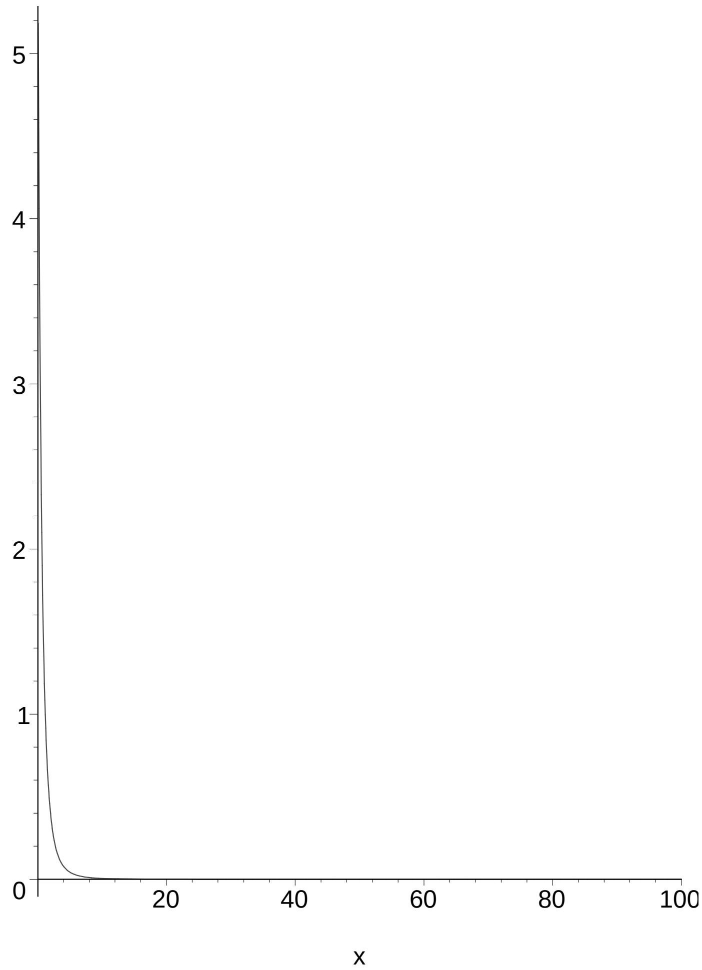

Figure 3: Percent difference between the function and its approximation, $\sigma p = 1$.

We compare the approximate potential with $V(x)$ in figures 3 and 4, for different values of particle label σ. As is clear from these figures, the approximate potential follows $V(x)$ more closely the larger σ is and the further x is chosen from the horizon ($x = 0$). Remarkably, even for $\sigma p = 1$ the potentials only deviate by a few percent right at the horizon, where the fractional deviation becomes $(V - V_{\rm approx})/V \to 1/(16p^2\sigma^2 + 1)$. For this reason we believe the approximate potential to more accurately capture the form of the Klein Gordon solutions near the horizon than would be possible using only an asymptotic expansion of the solutions in powers of $1/(\sigma p)$.

Using the approximate potential the Klein Gordon equation becomes

$$\frac{d^2F(x)}{dx^2} + \left(1 + \frac{4p}{x} + \frac{4p^2}{x^2}\right)\sigma^2\ F(x) = 0\,, \tag{3.10}$$

which can be solved exactly to give Whittaker functions [13] $M_{\chi,\mu}(z)$ and $M_{\chi,-\mu}(z)$, as the linearly-independent solutions. These are related to standard confluent hypergeometric

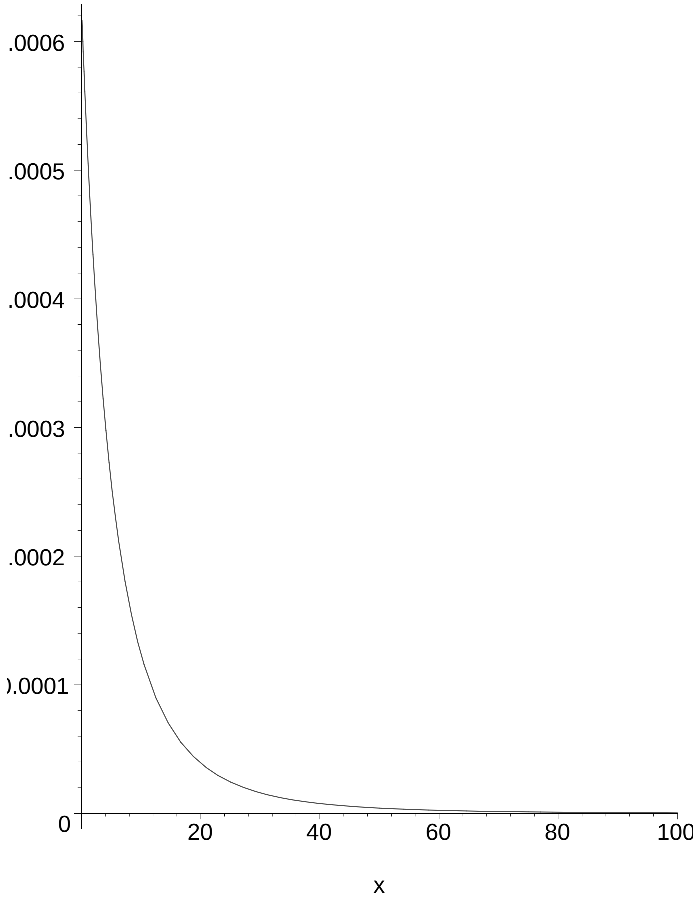

Figure 4: Percent difference between the function and it's approximation, $\sigma p = 100$.

functions according to

$$M_{\chi,\mu}(z) = z^{\mu+\frac{1}{2}}\, e^{-z/2}\, {}_1F_1\left(\mu - \chi + \frac{1}{2}, 2\mu + 1; z\right). \tag{3.11}$$

The parameters χ, μ and z are given in terms of p, σ and x by the relations: $\chi = -2i\sigma p$, $z = 2i\sigma x$ and $\mu = i\mu_I$, with $\mu_I = \frac{1}{2}\sqrt{16\sigma^2 p^2 - 1}$. Notice that χ, μ and z are all pure imaginary so long as $|\sigma p| > 1/4$, as we shall assume in what follows.

For the purposes of a particle-production calculation we are interested in the combination of these functions which have positive and negative frequency near $x = 0$ and $x \to \infty$. It happens that it is $M_{\chi,-\mu}(z)$ which is positive frequency near $x = 0$ and $M_{\chi,\mu}(z)$ which is negative frequency, as may be seen from the small-z limit

$$M_{\chi,\mu}(z) = z^{\mu+1/2}\, [1 + \mathcal{O}(z)]\,. \tag{3.12}$$

The assignment of positive and negative frequencies follows once this singular part is re-expressed in terms of the time coordinate, x, in which case $z^{\mu+1/2} \propto x^{1/2} \exp[i\mu_I \log(x/p)]$. (Recall the standard phase convention calls $\exp[-i\varphi(t)]$ positive-frequency when $\varphi(t)$ increases with advancing time, such as for $\varphi = \omega t$ with $\omega > 0$.) For later purposes we also

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record here the useful identity:

$$[M_{\chi,-\mu}(z)]^* = M_{-\chi,\mu}(e^{-i\pi}z) = e^{-i\pi(\mu+1/2)}\, M_{\chi,\mu}(z)\,, \tag{3.13}$$

where we assume $|\sigma p| > 1/4$ in order to use that all three of χ, μ and z are pure imaginary.

For $x \to \infty$, on the other hand, it is the particular linear combination

$$W_{\chi,\mu}(z) = \frac{\Gamma(-2\mu)}{\Gamma(1/2-\mu-\chi)}\, M_{\chi,\mu}(z) + \frac{\Gamma(2\mu)}{\Gamma(1/2+\mu-\chi)}\, M_{\chi,-\mu}(z)\,, \tag{3.14}$$

which is positive frequency, as may be seen from its asymptotic form

$$W_{\chi,\mu}(z) \sim z^{\chi}\, e^{-z/2} \left[1 + \mathcal{O}\left(\frac{1}{z}\right)\right], \tag{3.15}$$

since $e^{-z/2} = e^{-i|\sigma|x}$. That it is the absolute value of σ which appears here follows from a careful treatment of the phase accumulated when z changes sign due to the branch cut at $z = 0$, using identities like eq. (3.13). The additional phase associated with the factor z^{χ} does not change this conclusion. For instance, for $\sigma p \gg 1$ its effect is simply to change $e^{-i|\sigma|x}$ to $e^{-i|\sigma|x_*}$, where $x_* = x + 2p\, \log(x/p)$ is the tortoise coordinate.

With these preliminaries we may now proceed with the particle-production calculation. If we start with the mode expansion which is appropriate for large x, we have

$$\Phi(x,r,\theta,\phi) = \sum_m \int dk\, d\sigma \left[a_{km\sigma}\; u_{km\sigma}(x,r,\theta,\phi) + \text{c.c.}\right], \tag{3.16}$$

where $a_{km\sigma}$ denotes the mode destruction operator and $u_{km\sigma}(x,r,\theta,\phi) \propto e^{i\sigma r}\, W_{\chi,\mu}(z)$.

On the other hand, near the horizon we instead have

$$\Phi(x,r,\theta,\phi) = \sum_m \int dk\, d\sigma \left[b_{km\sigma}\; v_{km\sigma}(x,r,\theta,\phi) + \text{c.c.}\right], \tag{3.17}$$

where $b_{km\sigma}$ are destruction operators and $v_{km\sigma}(x,r,\theta,\phi) \propto e^{i\sigma r}\, M_{\chi,-\mu}(z)$. Particle production occurs because the expansion of $u_{km\sigma}$ in terms of $v_{km\sigma}$ implies that $b_{km\sigma}$ can be expressed as a linear combination of $a_{km\sigma}$ and $a^*_{k,-m,-\sigma}$. (It is conservation of m and σ which permits only modes with opposite signs of m and σ to mix in this way.)

For simplicity it is convenient at this point to choose $k = m = 0$ and to suppress the k and m labels. The decomposition of b_σ in terms of a_σ and $a^*_{-\sigma}$ is found by choosing a particular $\sigma > 0$ and following those terms whose r-dependence is proportional to $e^{i\sigma r}$. Keeping in mind that $\chi(-\sigma) = -\chi(\sigma) = -\chi$, $z(-\sigma) = -z(\sigma) = -z$ and $\mu(-\sigma) = \mu(\sigma) = \mu$, and using eqs. (3.13) and (3.14) we find:

$$\begin{aligned} a_\sigma\, W_{\chi,\mu}(z) + a^*_{-\sigma}\left[W_{-\chi,\mu}(-z)\right]^* &= \\ &= \frac{\Gamma(2\mu)}{\Gamma\left(1/2+\mu-\chi\right)} \left[a_\sigma\, M_{\chi,-\mu}(z) + a^*_{-\sigma}\, e^{-i\pi(1/2-\mu)}\, M_{-\chi,-\mu}\left(e^{-i\pi}z\right)\right] \\ &\sim \frac{\Gamma(2\mu)}{\Gamma\left(1/2+\mu-\chi\right)}\, z^{1/2-\mu}\left[a_\sigma + a^*_{-\sigma}\, e^{-i\pi(1-2\mu)}\right]. \end{aligned} \tag{3.18}$$

The last line gives the asymptotic form near $x = 0$.

From these manipulations we see that the operators a_σ and b_σ are related to one another by:

$$b_\sigma = \Lambda(\sigma) \left[a_\sigma - e^{2i\pi\mu} \, a^*_{-\sigma} \right], \tag{3.19}$$

where $\Lambda^2 = 1/[1 - \exp(4i\pi\mu)]$ is determined from the normalization requirement $[b_\sigma, b^*_\sigma] = [a_\sigma, a^*_\sigma]$. Inverting this relation gives the expression

$$a_\sigma = \Lambda(\sigma) \left[b_\sigma + e^{2i\pi\mu} \, b^*_{-\sigma} \right], \tag{3.20}$$

which is the main result which is required for the particle-production calculation.

We imagine preparing the field in the ground state as seen by observers crossing the horizon: $b_\sigma|0\rangle = 0$, and then asking for the number of late-time particles which this state would contain. We find in this way our final result:

$$\langle N_\sigma \rangle = \langle 0| a^*_\sigma \, a_\sigma |0\rangle = \frac{1}{e^{4\pi\mu_I} - 1}, \tag{3.21}$$

where we recall $\mu_I = \frac{1}{2}(16\sigma^2 p^2 - 1)^{1/2}$.

This result is very suggestive of a thermal form. Indeed if we write $\omega = [\sigma^2 - 1/(4p)^2]^{1/2}$, so $\omega \approx \sigma$ when $\sigma p \gg 1$, then eq. (3.21) is *precisely* thermal,

$$\langle N_\sigma \rangle = \frac{1}{e^{8\pi\omega p} - 1} \tag{3.22}$$

if ω is interpreted as the particle energy. (This interpretation is natural near the horizon where $v_\sigma \sim z^{-\mu} \sim e^{-i\mu_I \log x} \sim e^{-2ip\omega \log x} \sim e^{-i\omega x_*}$ shows that ω is the eigenvalue of the operator $i\partial_{t_*}$.) Since our derivation assumes μ is pure imaginary it breaks down for $|\sigma p| < \frac{1}{4}$, where ω becomes imaginary. Our approximate form, $V_{\rm approx}$, also provides a worse description of the full result, V, for σ this small. Consequently we cannot yet say whether these modes are also pair produced at late times.

Even more remarkably, the corresponding temperature is

$$T = \frac{1}{8\pi p}, \tag{3.23}$$

which is exactly the same result obtained earlier by euclideanizing the metric in the static regions. *A priori* these did not have to agree since the euclidean calculation describes the particles seen by an accelerating, static observer behind the horizons, while in the present instance the temperature corresponds to the distribution of particles which are produced by the time-dependent fields in region I.

4. Discussion

The metrics studied here were proposed in ref. [6] with an eye to using their time dependence for cosmological applications. Clearly, the classical instability of the horizon towards singularity formation diminishes the cosmological impact of these solutions. In particular it prevents the passage from region III (past, contracting time-dependent solution) into region

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I (future, expanding time-dependent solution) in a way which does not hit a singularity. This result is consistent with the strong cosmic censorship conjecture, since the observer crossing the Cauchy horizon, decoupling from her past history, would otherwise find a naked singularity, from which information can come, and yet would be able to avoid the singularity. It would be interesting to see if there are cases that avoid this problem [12, 14].

A similarly wet blanket is thrown on the S-brane interpretation of this geometry in terms of a rolling tachyon field [1, 15]. In this interpretation we imagine the rolling of the tachyon field from one minimum of the potential at $t \to -\infty$ to the other minimum at $t \to \infty$. Each vacuum could then be identified with the asymptotic, flat infinite past and future of figure 1 respectively. The local maximum of the potential would then be identified with the horizon. Our result presents an obstruction to this realization of the rolling by making it impossible to miss a singularity in between. (This singularity problem is also shared by other proposed S-brane geometries.)

The existence of the singularity need not invalidate the interpretation of region I as describing the metric produced by the late-time rolling of a tachyon from a local maximum to a later local minimum, however, since this part can be described purely by the geometry in the future of these singularities. Following this interpretation, our result for the particle production could be relevant to the determination of particle production after tachyon condensation. This would be particularly interesting for the string scenarios of hybrid inflation from D-brane interactions, as proposed in [16]. In this case the particle production computed here could lead to the determination of the reheating after inflation, although the explicit connection between the rolling tachyon potential and the S-brane geometries is still not fully understood. For a recent discussion of reheating from tachyon condensation see [17].

We find our particle-production result to be intriguing in its own right, due to the thermal character of the produced particles, and the connection which it indicates between the temperature of this distribution and the temperature obtained by euclideanizing the metric's static region. This connection is all the more intriguing given the classical instability which we find, which is likely to convert the intervening horizons into curvature singularities.

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